UNIT-1

(Lecture-3)

Realization of Digital Systems:

Direct form realization of IIR systems

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Basic IIR Digital Filter Structures

- The causal IIR digital filters we are concerned with in this course are characterized by a real rational transfer function of z^{-1} or, equivalently by a constant coefficient difference equation
- From the difference equation, it can be seen that the realization of the causal IIR digital filters requires some form of feedback

Basic IIR Digital Filter Structures

- An N-th order IIR digital transfer function is characterized by $2N+1$ unique coefficients, and in general, requires $2N+1$ multipliers and $2N$ two-input adders for implementation
- **Direct form IIR filters:**
  Filter structures in which the multiplier coefficients are precisely the coefficients of the transfer function

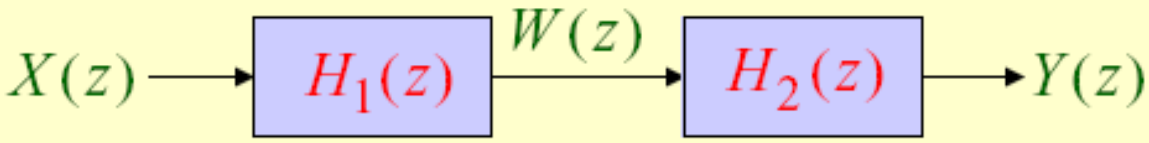
Direct Form IIR Digital Filter Structures

- Consider for simplicity a 3rd-order IIR filter with a transfer function

$$H(z)=\frac{P(z)}{D(z)}=\frac{p_0+p_1z^{-1}+p_2z^{-2}+p_3z^{-3}}{1+d_1z^{-1}+d_2z^{-2}+d_3z^{-3}}$$

- We can implement $H(z)$ as a cascade of two filter sections as shown on the next slide

Direct Form IIR Digital Filter Structures



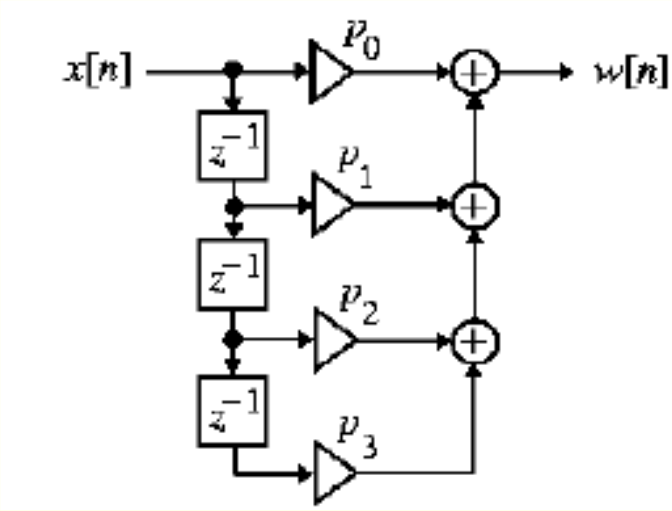
where

$$H_1(z) = \frac{W(z)}{X(z)} = P(z) = p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3}$$

$$H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{D(z)} = \frac{1}{1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}}$$

Direct Form IIR Digital Filter Structures

- The filter section $H_1(z)$ can be seen to be an FIR filter and can be realized as shown below



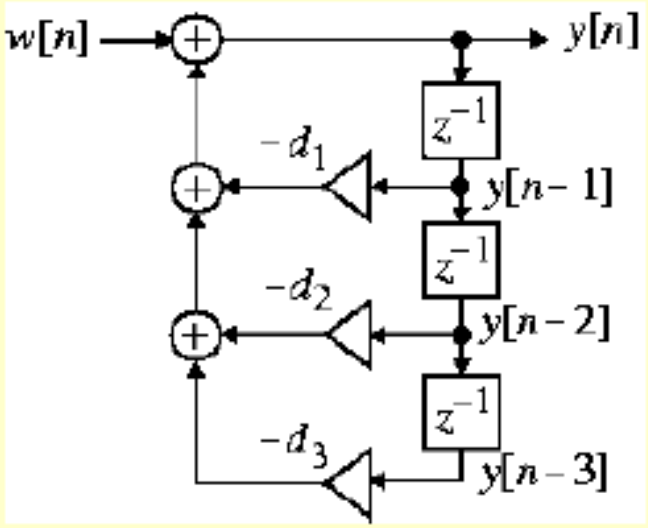
$$w[n] = p_0 x[n] + p_1 x[n-1] + p_2 x[n-2] + p_3 x[n-3]$$

Direct Form IIR Digital Filter Structures

- The time-domain representation of $H_2(z)$ is given by

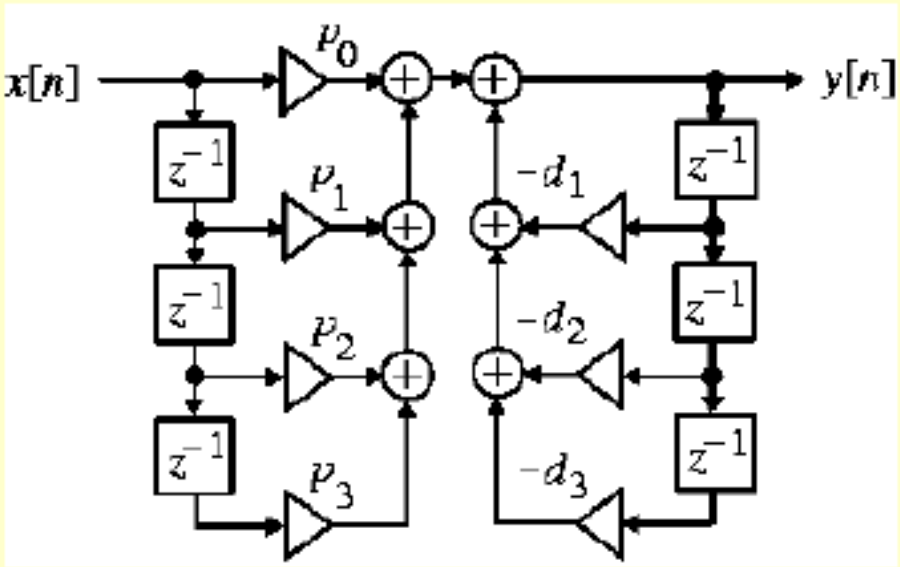
$$y[n] = w[n] - d_1 y[n-1] - d_2 y[n-2] - d_3 y[n-3]$$

- The realization of $H_2(z)$ follows from the above equation and is shown in the figure



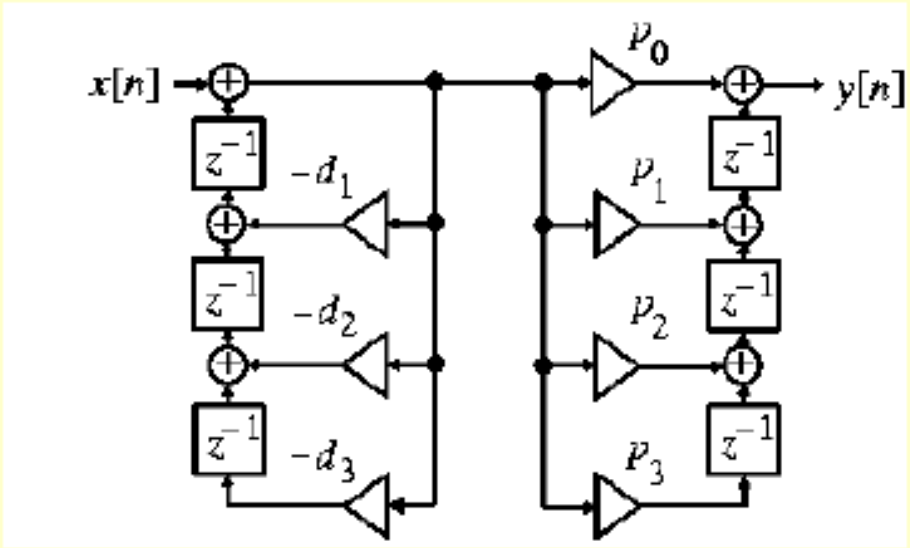
Direct Form IIR Digital Filter Structures

- A cascade of the two structures realizing $H_1(z)$ and $H_2(z)$ leads to the realization of $H(z)$ shown below and is known as the **direct form I** structure



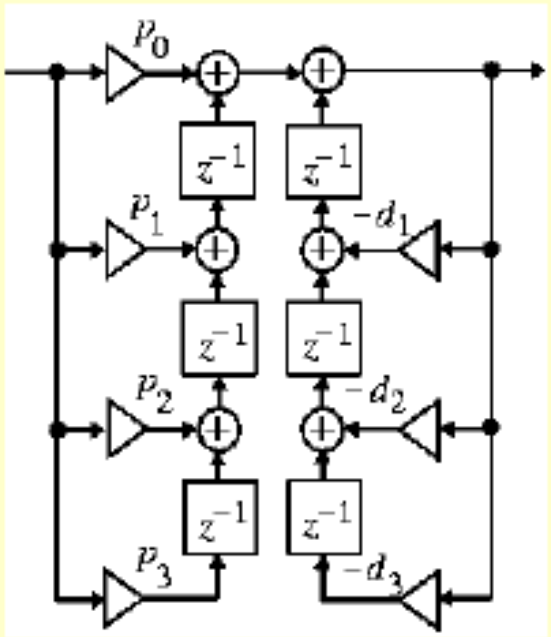
Direct Form IIR Digital Filter Structures

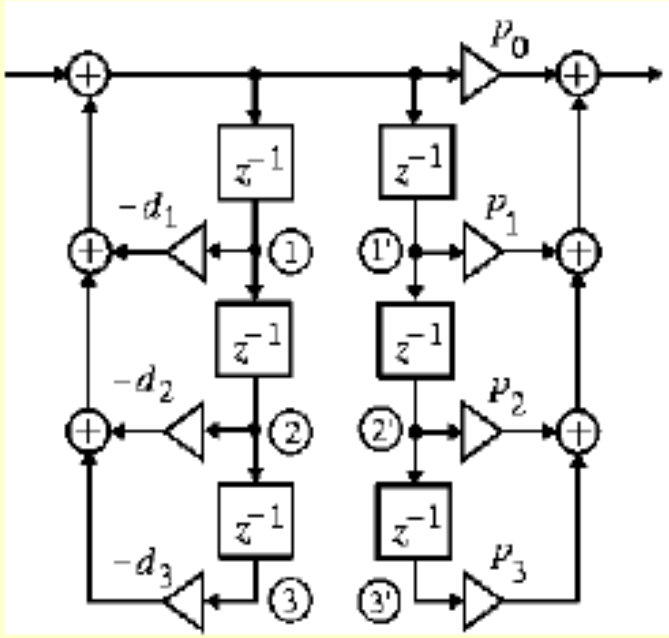
- The direct form I structure is noncanonic as it employs 6 delays to realize a 3rd-order transfer function
- The transpose of the direct form I structure is shown in the figure and it is called the **direct form I_t** structure



Direct Form IIR Digital Filter Structures

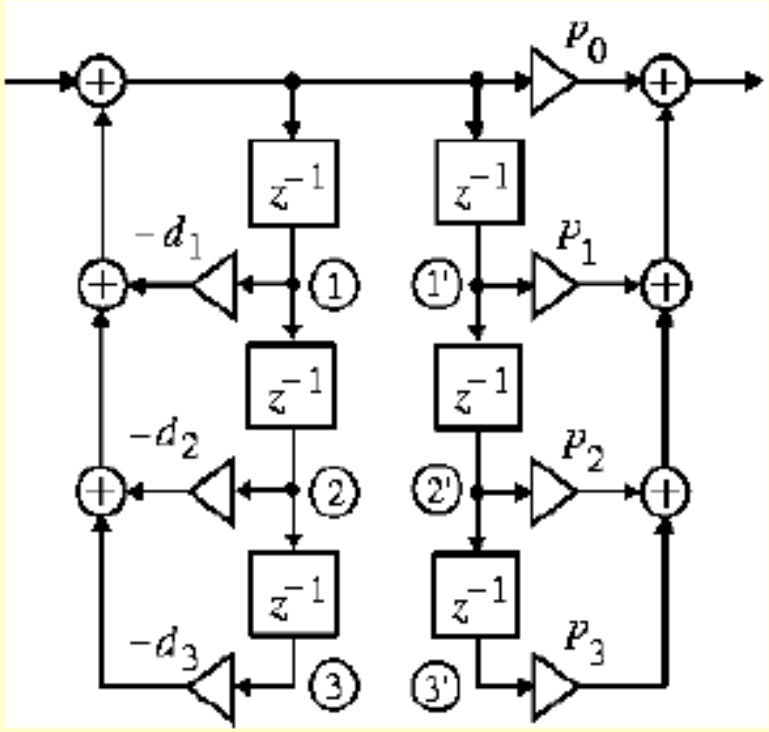
- Various other noncanonic direct form structures can be derived by simple block diagram manipulations as shown below





Direct Form IIR Digital Filter Structures

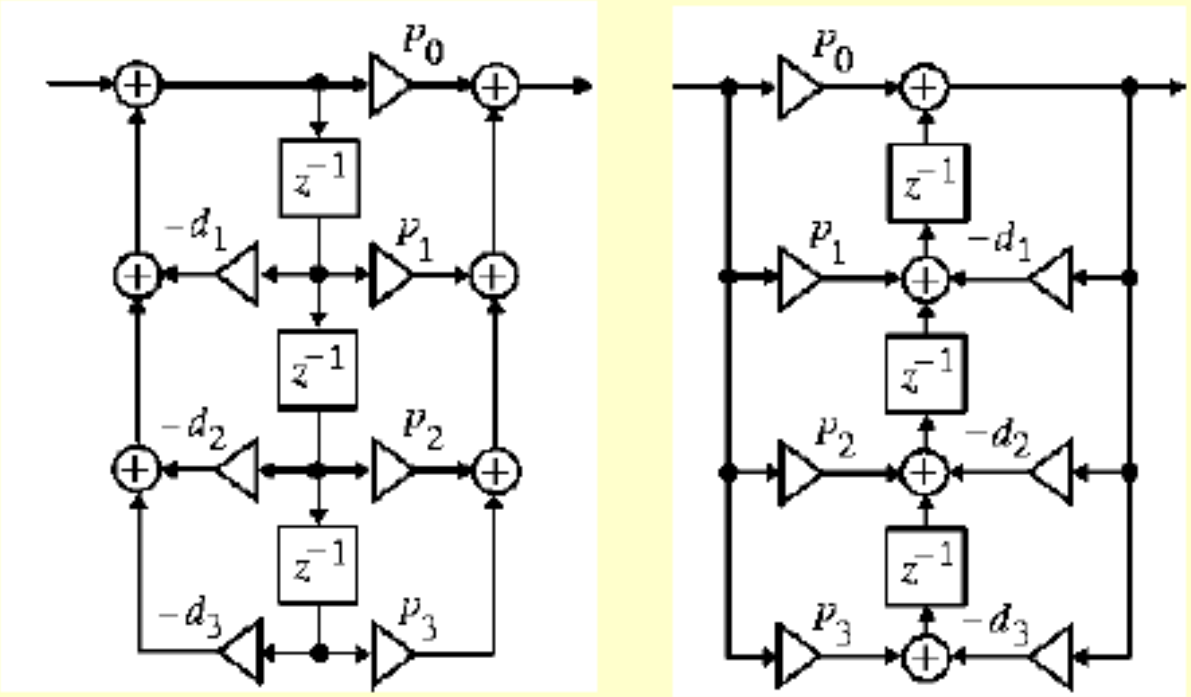
- Observe in the direct form structure shown right, the signal variable at nodes ① and ①' are the same, and hence the two top delays can be shared



Direct Form IIR Digital Filter Structures

- Likewise, the signal variables at nodes ② and ②' are the same, permitting the sharing of the middle two delays
- Following the same argument, we can share the bottom two delays leading to the final canonic structure, which is called the **direct form II** structure
- The direct form II and the **direct form II$_t$** structure are shown on the next slide

Direct Form IIR Digital Filter Structures



- Direct form realizations of an N-th order IIR transfer function should be evident